Discrete harmonic analysis and applications to ergodic theory

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Abstract

Given $d, k \in \mathbb{N}$, let P_j be an integer-valued polynomial of k variables for every $1 \le j \le d$. Suppose that $(X, \mathcal{B}, \mu)$ is a σ-finite measure space with a family of invertible commuting and measure preserving transformations $T_1, T_2, \ldots, T_d$ on X. For every $N \in \mathbb{N}$ and $x \in X$ we define the ergodic Radon averaging operators by setting

$$A_N f(x) = \frac{1}{N^k} \sum_{m \in [1,N]^k \cap \mathbb{Z}^k} f(T_1^{P_1(m)} \circ T_2^{P_2(m)} \circ \ldots \circ T_d^{P_d(m)} x).$$

We will show that for every $p > 1$ and for every function $f \in L^p(X, \mu)$, there is a function $f^* \in L^p(X, \mu)$ such that

$$\lim_{N \to \infty} A_N f(x) = f^*(x)$$

μ-almost everywhere on X. We will achieve this by considering r-variational estimates.